Protection Electrode Signal Processing Algorithm

(For The PIP-II Injector Test at Fermilab)

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Abstract

Both 50 Ohm and 200 Ohm PIP2-IT MEBT kickers have a pair of protection electrodes to protect the kicker plates from beam scraping. This paper derives a digital processing algorithm intended to protect the electrode from damage in real-time. It is the equivalent of the integral expression of current striking the electrodes as described in the FRS document PIP-II Injector Test MEBT Kicker Protection Electrode Electronics, ED0001304 Rev A, dated 2/17/2017. This paper explains the derivation of the discrete time equivalent of the analytical expression as well as an approach to minimize computational FPGA resources allowing the algorithm to calculate fast enough to enable a protection system to respond in microseconds.

Problem Definition

Defined in the FRS, A. Shemyakin proposed [1] the following criteria eq. (1) for the Machine Protection System (MPS) to interrupt beam from excessive current striking a kicker protection electrode (see also [2]), where $I(t_1)$ is measured protection electrode current independent of whether beam is pulsed or CW:

$$Q(t) = \int_0^t I(t_1) \exp\Big[-\frac{t-t_1}{\tau}\Big] dt_1 = e^{-t/\tau} \int_0^t I(t_1) e^{t_1/\tau} dt_1. \tag{1}$$

$Q(t)$, expressed in $A{\cdot}s$, is the integral of beam current representing a quantity proportional to the energy causing electrode heat buildup. It includes a term, τ, defining the rate at which this energy is dissipated by conduction and radiation. Even though eq. (1) is merely a best-guess, it represents real-time thermal buildup behavior so that continuously comparing computed $Q(t)$ with a defined threshold limit allows for a fast protection response.

The FRS set $\tau = 10\ ms$ and the MPS should interrupt the beam when $Q(t) > 50\ mA \cdot \mu s$ this later will be referred to as the trip level. One should note that integration starts from zero, which means that the beam and the protection electronics are turned on at the same time. For example, lets assume that the Machine is operating at CW mode and 5 μA of average beam current is scraping the protection electrode. In this situation the processed Electrode Signal $Q(t)$ according to eq.(1) will represent in the first tens of milliseconds an exponentially rising edge with rise time of 10 ms with maximum amplitude of $Q(t) = 50\ mA{\cdot}\mu s$, see fig. 1. As it can be seen this is normal operation mode and MPS will not stop the Machine in this case, because $Q(t)$ never exceeds 50 $mA \cdot \mu s$.

For another arbitrary example lets assume that the Machine is operating in some mode when 0.2 mA of beam is scraping the protection electrode with 300 Hz rep-rate

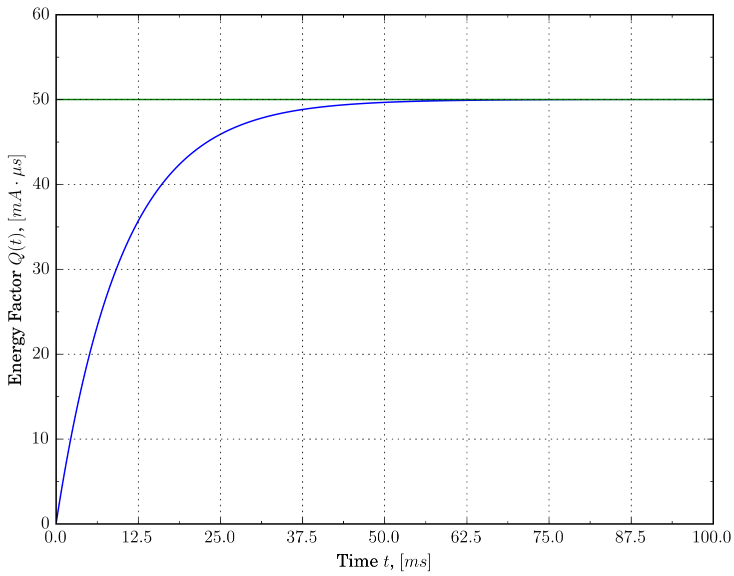

Figure 1: 5 μA scraping electrode CW, $Q(t)$ calculated using eq. (1)

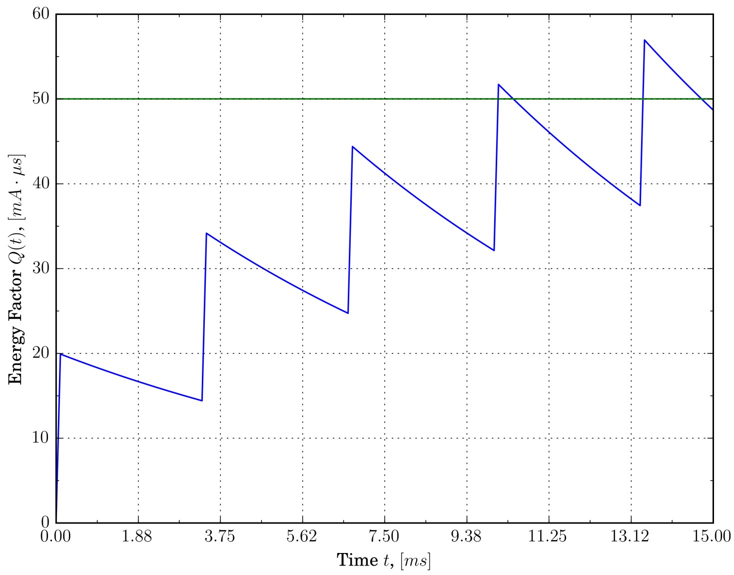

Figure 2: 2 mA, 100 μs, 300 Hz pulses scraping electrode , $Q(t)$ calculated using eq. (1)

and 100 μs pulse width. In this case MPS will stop the Machine in approximately 10 ms, see fig. 2.

Integral eq. (1) can be calculated using various engineering techniques. But rather than designing new electronics for that it seems that the most convenient approach is to use the Instrumentation Department's 125 MHz digitizers [3] already measuring the electrode current and do direct digital signal processing with the on-board FPGA.

Conversion to Discrete-time

Next convert eq. (1) to discrete form. If $I(t_1)$ is a square pulse with length T and magnitude I the integral (1) turns to eq. (2):

$$Q(T) = I\tau\Big(1 - e^{-T/\tau}\Big). \tag{2}$$

The electronics will receive some value of I (or some value directly proportional to I), process this data in real time and the result should be the value of Q. From this point of view the transfer function of electronics can be defined as eq. (3), where $\mathcal{L}$ is the Laplace transform:

$$k(s) = \frac{\mathcal{L}\big[Q(T)\big]}{\mathcal{L}\big[I\tau\big]}. \tag{3}$$

By definition, the denominator is (see [4]):

$$\mathcal{L}\big[I\tau\big] = \int_0^\infty I\tau e^{-sT} dT = I\tau\frac{1}{s}, \quad (I\tau = const). \tag{4}$$

The numerator is:

$$\begin{aligned}
\mathcal{L}\big[Q(T)\big] &= \mathcal{L}\Big[I\tau\Big(1 - e^{-T/\tau}\Big)\Big] = \int_0^\infty I\tau e^{-sT}\Big(1 - e^{-T/\tau}\Big) dT = \\
&= I\tau \int_0^\infty e^{-sT} dT - I\tau \int_0^\infty \exp\Big[-T\big(\frac{1}{\tau} + s\big)\Big] dT = \\
&= -\frac{1}{s} I\tau e^{-sT}\Big|_0^\infty + I\tau\frac{1}{\frac{1}{\tau} + s} \exp\Big[-T\big(\frac{1}{\tau} + s\big)\Big]\Big|_0^\infty = \\
&= I\tau\left[\frac{1}{s} - \frac{1}{\frac{1}{\tau} + s}\right].
\end{aligned} \tag{5}$$

Now from eq. (3) the transfer function of electronics can be found as eq. (6):

$$k(s) = \frac{I\tau\left[\frac{1}{s} - \frac{1}{\frac{1}{\tau}+s}\right]}{I\tau\frac{1}{s}} = \frac{1}{1 + \tau s}. \tag{6}$$

We can rewrite eq. (2) in s-domain instead of time domain by using eq. (3) and (6):

$$Q = I\tau\frac{1}{1 + \tau s}. \tag{7}$$

Next we apply the Z-transform to eq. (7) by using The Forward Difference Method (see [5]) assuming that $s = \frac{z-1}{\delta T} = fz - f$ where $f = \frac{1}{\delta T}$ is sampling frequency. As a result we get eq. (7) in Z-domain:

$$\begin{aligned}
&\mathcal{Z}[Q] = \mathcal{Z}[I]\frac{\tau}{1 + \tau(fz - f)}, \\
&\mathcal{Z}[Q]\Big(1 + \tau(fz - f)\Big) = \mathcal{Z}[I]\tau, \\
&\mathcal{Z}[Q] + \mathcal{Z}[Q]\tau fz - [Q]\tau f = \mathcal{Z}[I]\tau.
\end{aligned} \tag{8}$$

By using the rule that for some discrete function Y with $C = const$ in Z-domain (see [6]):

$$\mathcal{Z}[Y] \cdot z^k \cdot C = Y_{n+k} \cdot C, \tag{9}$$

we get the discrete difference equation of eq. (8):

$$Q_n + Q_{n+1}\tau f - Q_n \tau f = I_n \tau. \tag{10}$$

By replacing f with $\frac{1}{\delta T}$, $(n+1)$ with n and n with $(n-1)$ we finally get:

$$Q_n = \delta T \cdot I_{n-1} - \frac{\delta T}{\tau} Q_{n-1} + Q_{n-1}. \tag{11}$$

In fact eq. (11) is a discrete form of eq. (1) and it can be implemented in the FPGA to process the protection electrode current in a digital way and protect the electrode in real time.

Coefficient Scaling

Several techniques will be used to be assured the FPGA can perform computations fast enough for this application. First, all multiplications will be done with unsigned integers rather than floating point. This requires scaling-up coefficients in order that computed values be greater than zero but large enough to only preserve resolution. The chosen scaling factor is a binary number that allows a subsequent division in one of the integration terms to be simply a matter of extracting most significant bits and eliminate a time-costly real division operation. Using this technique, no division operations are performed in this algorithm. Also, coefficient values will be combined and pre-calculated to minimize the number of multiplications to perform.

The measured beam current is first converted to a voltage and then digitized with a 14-bit A/D converter. This digitizer is bipolar, so I_n magnitude is 13 bits having a scale factor $\frac{12\ mA}{1\ V}$ (see [7]). The ADC codes for 5 μA and 5 mA of current are obtained by applying the magnitude-scaling factor shown in eq. (12):

$$I^{/}_{5\mu A} = 5 \cdot 10^{-6} \cdot \frac{2^{13}}{12 \cdot 10^{-3}} = 3;\ I^{/}_{5mA} = 5 \cdot 10^{-3} \cdot \frac{2^{13}}{12 \cdot 10^{-3}} = 3413. \tag{12}$$

The values of both δT and τ in eq.(11) require scaling since they are small fractions. Both need to be scaled the same, because they appear as a ratio in one equation term, and their ratio must remain proportional. Suppose that the sampling frequency of the digitizer will be $\frac{125\ MHz}{8} = 15.625\ MHz$. In this case $\delta T = 64 \cdot 10^{-9}\ s$. Thus, the value chosen for the time-scaling factor is 2^{34}, and the new sampling time interval becomes:

$$\delta T = 64 \cdot 10^{-9} \cdot 2^{34} \approx 1100. \tag{13}$$

By using new scaled units from eq. (12) and (13) we can rewrite eq. (11) to the form of eq. (14).

$$\begin{aligned} Q_n &= 1100 \cdot I^{/}_{n-1} - \frac{1100}{\tau \cdot 2^{34}} Q_{n-1} + Q_{n-1} = \\ &= 1100 \cdot I^{/}_{n-1} - \frac{\mathscr{F} \cdot Q_{n-1}}{2^{34}} + Q_{n-1}. \end{aligned} \tag{14}$$

The value of τ will be a settable parameter as an estimation to match the real electrode thermal decay time. The selectable values of $\mathscr{F} = 1100/\tau$ are:

τ, $[ms]$	$\mathscr{F}$
5	220000
10	110000
15	73333
20	55000

Choosing a larger value of τ implies the electrode takes longer to loose its heat between repetitive bursts, because calculated values will decay more slowly as $I(t)$ is integrated up. Trip level of 50 $mA \cdot \mu s$ in new units can be found as eq. (15) by multiplying it by both magnitude-scaling (as in eq. (12)) and time-scaling (as in eq. (13)) factors:

$$Q^{/}_{trip} = 5 \cdot 10^{-3} A \cdot 10 \cdot 10^{-6} s \cdot \frac{2^{13}}{12 \cdot 10^{-3}} \cdot 2^{34} = 5.863 \cdot 10^{8}. \tag{15}$$

It is suitable to have trip level only a two-digit unsigned number. Due to convenience of division by binary numbers in the FPGA we can divide eq. (15) by 2^{24}. Finally scaled trip level of 50 $\mu s \cdot mA$ will be eq. (16):

$$Q^{/}_{trip} = 5 \cdot 10^{-3} A \cdot 10 \cdot 10^{-6} s \cdot \frac{2^{13}}{12 \cdot 10^{-3}} \cdot 2^{34} \cdot \frac{1}{2^{24}} \approx 35. \tag{16}$$

The value for trip level is also to be a settable parameter and the scaled values are:

Q_{trip}, $[mA \cdot \mu s]$	$Q^{/}_{trip}[scaled\ units]$
40	28
45	31
50	35
55	38

The VHDL code that calculates eq. (14) is shown in Listing 1 (10 ms time constant, 16.625 MHz integration sampling rate). VHDL simulator (ModelSim) output is shown in fig. 3. Conditions are the same as for plot in fig. 1. Another result of calculation with similar conditions as in fig. 2 is shown in fig. 4. Where fig. 1 and fig. 2 are floating point calculations, figs. 3 and 4 are the discrete-time response. This VHDL code shown in Listing 1 performs the following calculation equivalent to the eq. (14):

$$Q3_{(n)} = Q2_{(n-1)} - \frac{Q4_{(n-1)}}{2^{34}} + Q3_{(n-1)}. \tag{17}$$

In this equation subscripts (n) and $(n-1)$ denote the way sequential statements are executed in VHDL.

Conclusion

The proposed eq.(14) should allow to digitally process protection electrode current in real time with the electronics that is already used in the PIP2-IT. No additional hardware development for signal processing is required. However, measurement of 5 μA still remains a challenge, because at such low current A/D converter will have only 3 levels of resolution. Development of more sensitive current measurement instrument may be required for the future CW operation. Real beam tests should be done to answer if the existing instrument will fit the Protection Electrodes Electronics FRS. To ideally fulfill these requirements an instrument with the dynamic range of $20log(10\ mA/1\ \mu A) = 80\ dB$ is required.

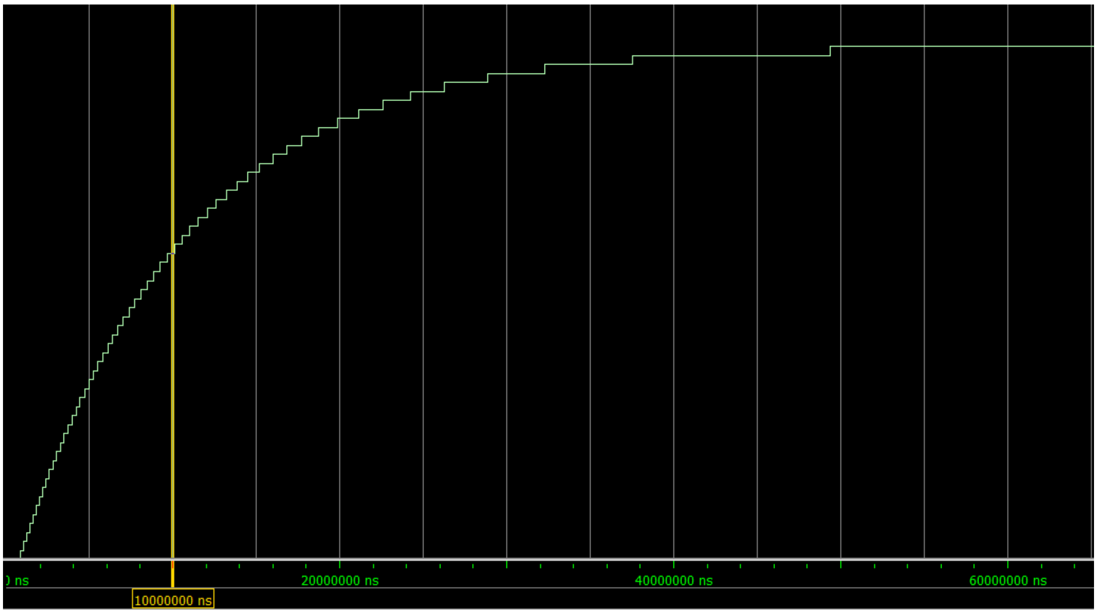

Figure 3: Calculation of Q_n with eq. (14), digitized version of fig. 1

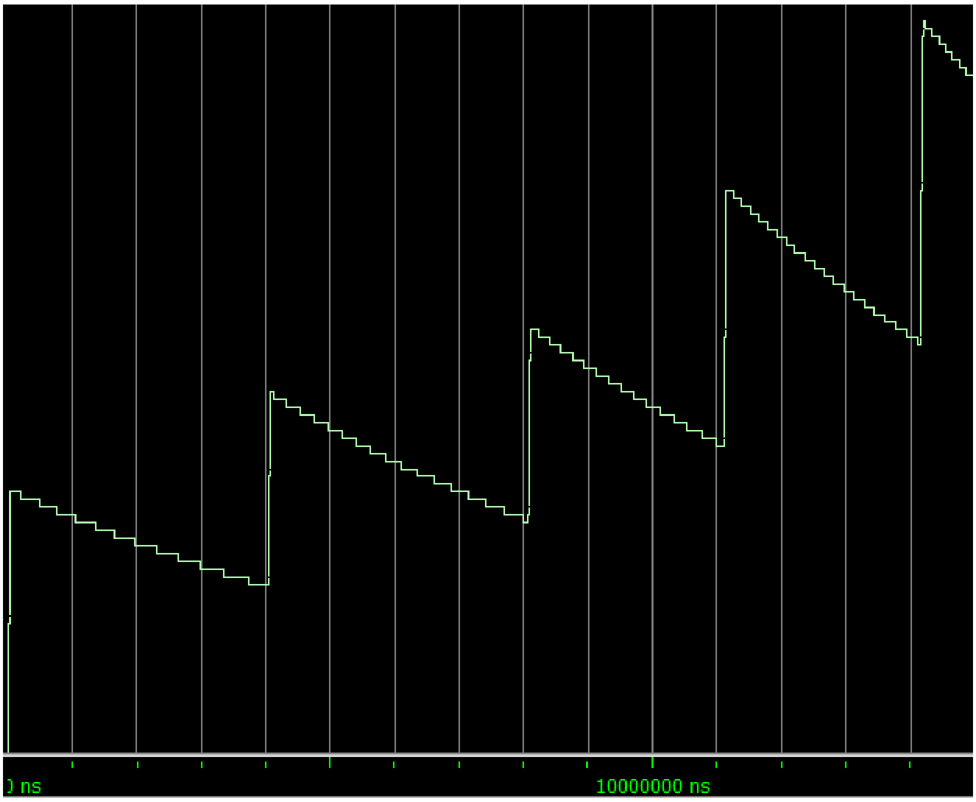

Figure 4: Calculation of Q_n with eq. (14), digitized version of fig. 2

Listing 1

```
LIBRARY IEEE;
USE IEEE.STD_LOGIC_1164.ALL;
USE IEEE.NUMERIC_STD.ALL;

ENTITY INTEGRATOR IS
PORT    (
    CLK         : IN STD_LOGIC;
    Q_OUT       : OUT STD_LOGIC_VECTOR(6 DOWNTO 0)  := (OTHERS => '0');
    I_IN        : IN STD_LOGIC_VECTOR(12 DOWNTO 0)
        );
END INTEGRATOR;

ARCHITECTURE BEHAVIORAL OF INTEGRATOR IS

    SIGNAL I1   : UNSIGNED(12 DOWNTO 0) := (OTHERS => '0');
    SIGNAL Q2   : UNSIGNED(23 DOWNTO 0) := (OTHERS => '0');
    SIGNAL Q3   : UNSIGNED(30 DOWNTO 0) := (OTHERS => '0');
    SIGNAL Q4   : UNSIGNED(47 DOWNTO 0) := (OTHERS => '0');
    CONSTANT dT : UNSIGNED(10 DOWNTO 0) := "10001001100"; -- 1100
    CONSTANT F  : UNSIGNED(16 DOWNTO 0) := "11010110110110000"; -- 1100/10ms

BEGIN
    PROCESS(CLK) BEGIN
        IF RISING_EDGE(CLK) THEN
            I1      <= UNSIGNED(I_IN);
            Q2      <= I1 * dT;
            Q3      <= Q2 + Q3 - Q4(47 DOWNTO 34);
            Q4      <= Q3 * F;
            Q_OUT   <= STD_LOGIC_VECTOR(Q3(30 DOWNTO 24));
        END IF;
    END PROCESS;
END BEHAVIORAL;
```

References

[1] PIP-II Document 223-v1 *PIP2IT MEBT Kicker Protection Electrode Electronics FRS.* http://pip2-docdb.fnal.gov:8080/cgi-bin/ShowDocument?docid=223

[2] PIP-II Document 135-v3 *The 200 Ohm Kicker Protection Electrodes Beam Signals Levels Estimation.* http://pip2-docdb.fnal.gov:8080/cgi-bin/ShowDocument?docid=135

[3] PIP-II Document 88-v1 *PXIE Beam Instrumentation Status Update (slide 6).* http://pip2-docdb.fnal.gov:8080/cgi-bin/ShowDocument?docid=88

[4] *Standard Mathematical Tables. 18th ed.* The Chemical Rubber Co., 1970, p.490, eq.(1)

[5] P. R. Vadhavkar *Mapping Controllers From the S-domain to the Z-domain Using Magnitude and Phase Invariance Methods: M.S. Thesis.* 2007, p.7, eq.(2.13) http://soar.wichita.edu/bitstream/handle/10057/1564/t07116.pdf?sequence=1

[6] J. G. Proakis, D. G. Manolakis *Digital Signal Processing: Principles, Algorithms and Applications. 2nd ed.* Macmillan Publishing Co., 1992, p.265, eq.(4.4.11)

[7] PIP-II Document 21-v1 *Update of PXIE Beam Instrumentation (slide 4).* http://pip2-docdb.fnal.gov:8080/cgi-bin/ShowDocument?docid=21