Suggestion for the beam jitter analysis procedure

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The beam jitter in the MEBT is still an issue, and we need to identify its source to eventually eliminate it. In the presentation at PIP-II Technical meeting by V.L.S. Sista (September 12, 2017), the source was identified to be in the LEBT, likely upstream of Solenoid #2. However, attempts to find the source by observing e.g. the correctors current signals (L. Prost presentation at the Operation meeting on October 5, 2017) did not succeed.

It is not clear whether a direct simulation of trajectories from the ion source through the LEBT and RFQ into the MEBT can provide the accuracy necessary for identifying the jitter source. An alternative approach is to compare the noise analysis directly with differential trajectories.

In measurements on April 23, 2018 (PM), the beam noise in MEBT BPMs was recorded along with differential trajectories in response to LEBT correctors 10 and 20, Ion source voltage and correctors, and the bend current. This note formulates the analysis procedure.

1. Optical simulation data
   a. Simulate with OptiM the response of the trajectory in the MEBT to each of 4 orthogonal initial vectors at the RFQ vanes $R0_i=(x0_i, xp0_i, y0_i, yp0_i)$. The natural choice is
      i. $R0_1=(1,0,0,0)$, $R0_2=(0,1,0,0)$, $R0_3=(0,0,1,0)$, $R0_4=(0,0,0,1)$
   b. Save the response in the BPM_j channel to $R0_i$ initial condition as $R_{i,j}$ (j runs through all X and Y channels).
2. Beam noise data
   a. Take BPM data from D44. Subtract averages.
   b. Convert the data set into a matrix with the time dependence of each BPM channel as a column.
   c. Apply SVD to the matrix. Look at dominant eigenvalues. All further work is only with several corresponding eigenvectors. Each eigenvector (*k-th)* has all available X and Y components, $B_{k,j}$.
   d. Fit each eigenvector to a linear combination of trajectories originated by 4 different sets of initial conditions $\{\overrightarrow{R0_i}\}$ with coefficients $CB_{k,i}$.
      i. Set the error sum as $F_{err}\left(\overrightarrow{CB_k}\right) = \sum_j \left(B_{k,j} - \sum_{i=1}^{4} CB_{k,i} \cdot R_{i,j}\right)^2$
      ii. Find for each eigenvector the 4-vector $\overrightarrow{CB_k}$ minimizing the error sum.
      iii. Calculate the relative residual error as $S_{err\_k} = \sqrt{\frac{F_{err}\left(\overrightarrow{CB_k}\right)}{\sum_j \left(B_{k,j}\right)^2}}$. The errors should be small to proceed with fitting.
3. Differential trajectory data

a. Fit each differential trajectory (*n-th*) $V_{n,j}$ to a linear combination of trajectories originated by 4 different sets of initial conditions $\{\overrightarrow{R0_i}\}$ with coefficients $CV_{n,i}$.
   i. Set the error sum as $FV_{err}\left(\overrightarrow{CV_n}\right) = \sum_j\left(V_{n,j} - \sum_{i=1}^4 CV_{n,i} \cdot R_{i,j}\right)^2$
   ii. Find for each trajectory the 4-vector $\overrightarrow{CV_n}$ minimizing the error sum.
   iii. Calculate the relative residual error as $SV_{err\_n} = \sqrt{\frac{FV_{err}\left(\overrightarrow{CV_n}\right)}{\sum_j\left(V_{n,j}\right)^2}}$. The errors should be small to use the trajectory for further fitting.

4. Try to find the jitter source from the list of differential trajectories sources
   a. Calculate the angles between coefficient 4-vectors for eigenvalues and for differential trajectories $\cos\theta_{k,n} = \frac{\overrightarrow{CB_k} \cdot \overrightarrow{CV_n}}{\left|\overrightarrow{CB_k}\right|\left|\overrightarrow{CV_n}\right|}$, where the standard definitions are used for the scalar product $\overrightarrow{CB_k} \cdot \overrightarrow{CV_n} = \sum_{i=1}^4 CB_{k,i} \cdot CV_{n,i}$ and vector length $\left|\overrightarrow{CB_k}\right| = \sqrt{\sum_{i=1}^4\left(CB_{k,i}\right)^2}$.
   b. List the angles and look for collinear vectors, i.e. cases with $\theta_{k,n} \approx 0$.
   c. In a case of success, compare corresponding data by eye. Also, calculate the error of the best fit to the differential trajectory $S_{err_{k,n}} = \sqrt{\frac{\sum_j\left(B_{k,j} - C_{k,n}V_{n,j}\right)^2}{\sum_j\left(B_{k,j}\right)^2}}$ after minimizing the sum by variation of the coefficient $C_{k,n}$. If $S_{err_{k,n}}$ is comparable with S_{err_k} and SV_{err_n}, $\frac{S_{err_{k,n}}}{\sqrt{S_{err\_k}{}^2 + SV_{err\_n}{}^2}} \sim 1$, the likely source of the jitter is found.

5. If there is no success, an alternative long way is to use the LEBT optics model.
   a. Calculate angles between 4-vectors of coefficients for the differential trajectories.
   b. Chose 4 of them that are far from being collinear (i.e. $\theta_{m,n}$ far from zero for each pair)
   c. For each eigenvector's $\overrightarrow{CB_k}$, find a linear combination of 4 corresponding differential trajectories' $\overrightarrow{CV_n}$, $\overrightarrow{CB_k} = \sum_{m=1}^4 C_{k,m} \cdot \overrightarrow{CV_m}$
      i. $\overrightarrow{C_k} = CV^{-1} \cdot \overrightarrow{CB_k}$, where CV is the matrix formed by vectors $\overrightarrow{CV_m}$.
   d. Assuming that is done with the LEBT correctors, put in simulation into each m-th corrector the current proportional to $C_{k,m}$.
   e. Find the trajectory parameters at the exit of the LEBT.
   f. Invert the trajectory from there, and pass it through the LEBT (with correctors off). The location where the trajectory crosses the axis is the location of the jitter source.