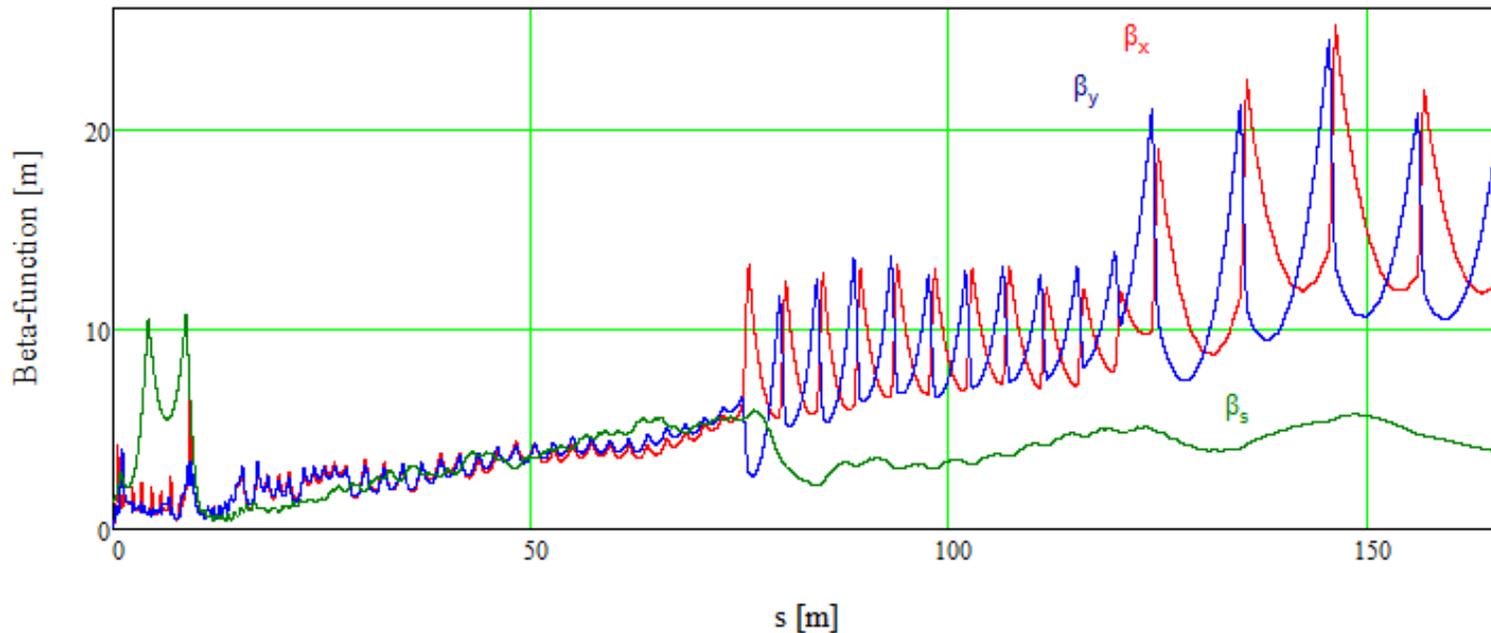


Requirements to PIP-II Alignment

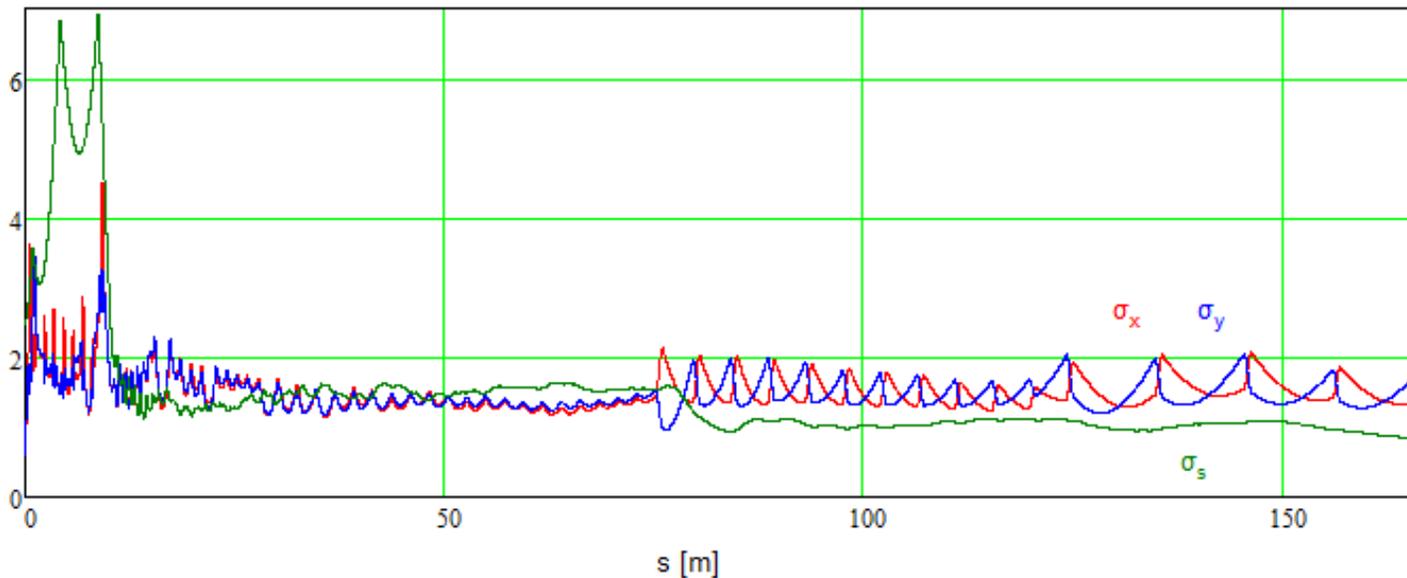
Valeri Lebedev

**PIP-II meeting
February 1, 2016
Fermilab**

Beam Sizes in the Linac



Rms beam sizes [mm]



$$\varepsilon_{xn} = \varepsilon_{yn} = 0.23 \mu\text{m}$$

$$\varepsilon_{xn} = \varepsilon_{yn} = 0.28 \mu\text{m}$$

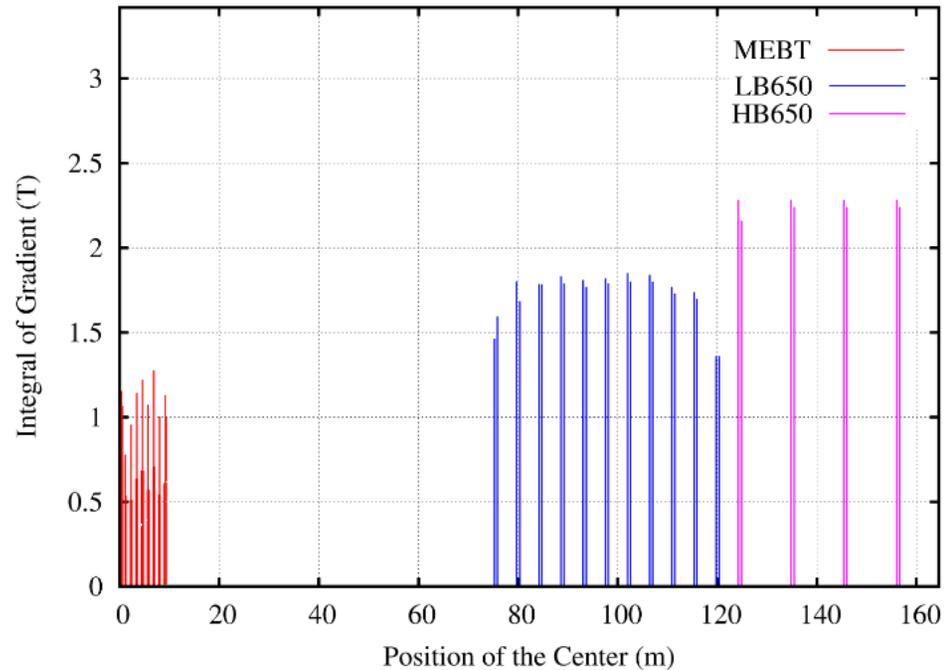
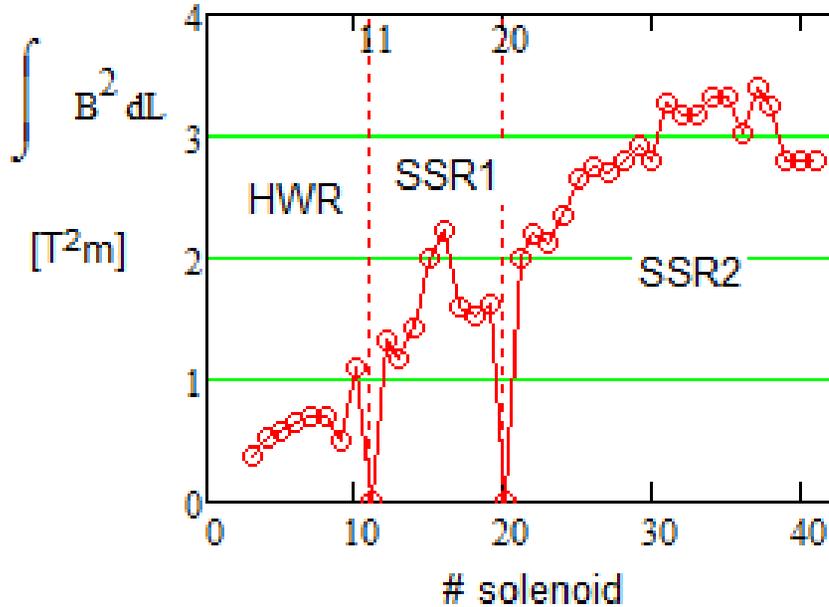
$$I_{\text{RFQbeam}} = 5 \text{ mA}$$

Growth of beta-functions is compensated by adiabatic damping:

$$\beta_{x,y} \propto \beta \gamma \propto p$$

An oscillation excited by lens displacement will stay with approximately the same amplitude to the linac end

Focusing Strength & Periodicity



Magnetic field of focusing solenoids (left) and integral strength of quadrupoles (right) along the linac.

■ Focusing strength of solenoid:
$$\frac{1}{F} = \frac{e^2 B_s^2 L_s}{4 p^2 c^2} \propto \frac{1}{(\beta\gamma)^2}$$

	HWR	SSR1	SSR2
Initial/final energy, MeV	2.1	10.5	35/185
Period (distance between solenoids), m	0.68	1.25	2.25
β -function at initial energy, m	1.1	2.7	3.3
Number of solenoids	8	8	21

Simple Estimates of the Betatron Motion Excitation

$$\begin{pmatrix} 1 & 0 \\ -\frac{\Phi}{2} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -\frac{\Phi}{2} & 1 \end{pmatrix} = \begin{pmatrix} 1 - \frac{L \cdot \Phi}{2} & L \\ \frac{L \cdot \Phi^2}{4} - \Phi & 1 - \frac{L \cdot \Phi}{2} \end{pmatrix}$$

$$\Rightarrow \beta = \sqrt{\frac{L \cdot F}{1 - \frac{L}{4 \cdot F}}} \quad \cos(\mu) = 1 - \frac{L}{2 \cdot F} \quad \frac{L}{F} = 4 \cdot \sin\left(\frac{\mu}{2}\right)^2$$

Amplification factor for oscillation excited by lens displacement

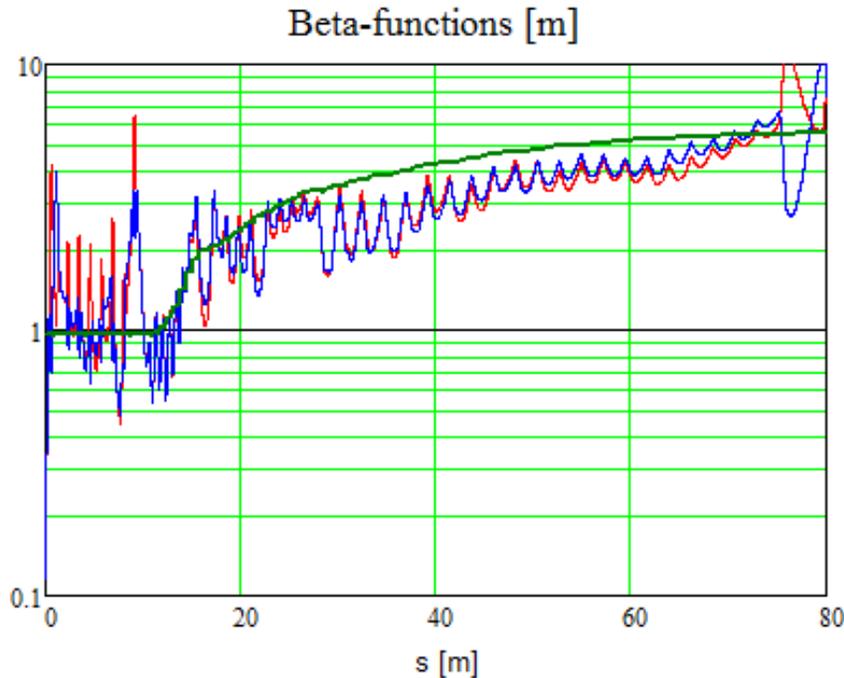
$$a = \beta \cdot \frac{y}{F} = y \cdot \sqrt{\frac{L}{F \cdot \left(1 - \frac{L}{4 \cdot F}\right)}} = y \cdot \frac{\sqrt{\left(4 \cdot \sin\left(\frac{\mu}{2}\right)\right)^2}}{\sqrt{1 - \sin\left(\frac{\mu}{2}\right)^2}} = 2 \cdot \tan\left(\frac{\mu}{2}\right) \cdot y$$

\Rightarrow amplification factor is: $A = 2 \cdot \tan\left(\frac{\mu}{2}\right)$

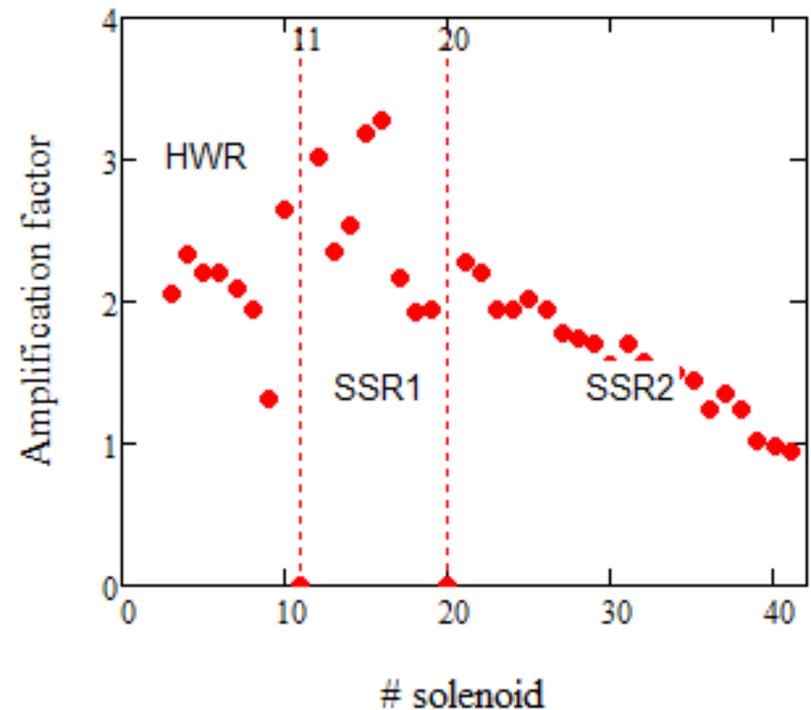
- Each section starts with ~90 deg. betatron phase advance
 \Rightarrow Amplification factor is about 2
- Real value of the amplification should be larger because lenses are stronger to compensate for defocusing coming from the space charge and cavities

Amplification Estimate for Real Focusing Strength

- For estimate we assume that $\beta_{x,y} = \beta_{x0} p (1 - 0.045 p / p_0)$, $\beta_{x0} = 1$ m, $p_0 = 2.1$ MeV, and energy growth is uniformly distributed among each cavity type



$$y_{\max} = \theta \beta = \frac{y_{\text{lens}}}{F} \beta \Rightarrow A = \frac{\beta}{F}$$



- This β -function parameterization does not account increased values of beta-functions at HWR-SSR1 transition. It makes an increase of ~ 1.5 to $A \sim 4$
- In further estimates we assume that the amplification factor is equal to 2.5 for all solenoids

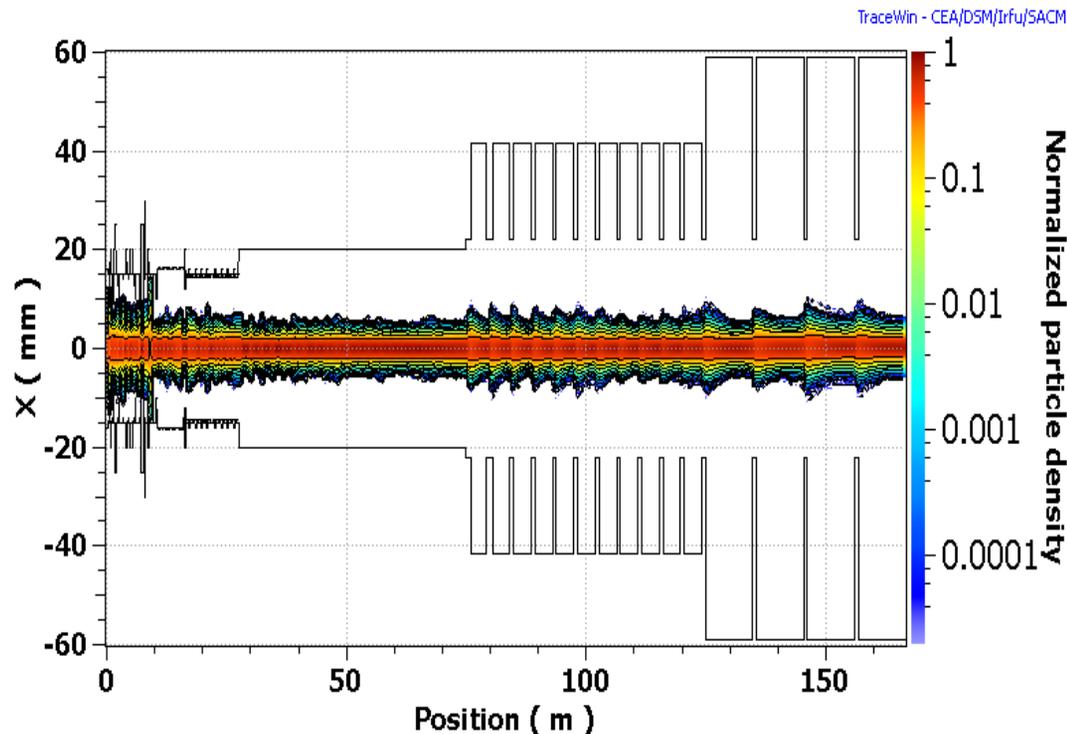
Alignment Requirements for Solenoids

- Let's require maximum betatron amplitude increase of 3.5 mm rms (9 mm maximum) after passing 8 solenoids (1 HWR or 2 SSR1 cryomodules) and assume that displacements are uncorrelated

- Then the single solenoid rms misalignment is

$$\Delta y_{sol} = \frac{3.5}{\sqrt{8}} \frac{1}{2.5} \approx 0.5 \text{ mm (rms)} \rightarrow \approx 1.25 \text{ mm (maximum)}$$

- Such requirement significantly simplifies initial steering through cryo-module and minimizes the beam loss inside cryomodule in the course of steering
- These requirements are set for the HWR and SSR1 in FRSs
 - ◆ It would be reasonable requirement for the SSR2



Present FRS Requirements to Angle and Offset Errors

	HWR	SSR1	SSR2
Solenoid coordinate alignment, rms, mm	0.5	0.5	0.1
Solenoid angular alignment, rms, mrad	1	1	0.5
Effective coordinate error due to angle, rms, mm	0.68	1.06	1.13
Integral solenoid strength, B^2dL , T^2m	2	4	5
Integral corrector strength, BdL , $mT \cdot m$	2.5	2.5	5

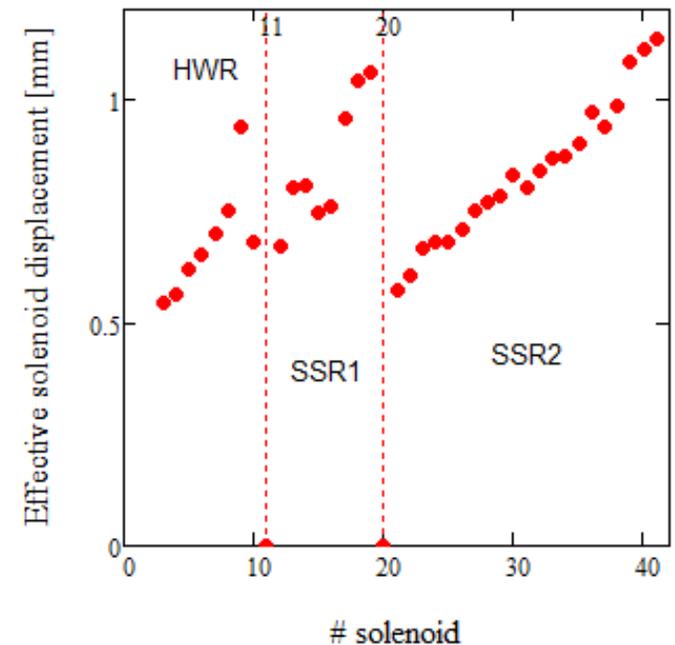
- Sensitivity to angle errors is significantly higher than to offset errors

- The angular amplification factor is:

$$A_{\theta} = 2\sqrt{F / L_{sol}}$$

- ⇒ So that the effective solenoid displacement corresponding to an angular error (ϕ) is:

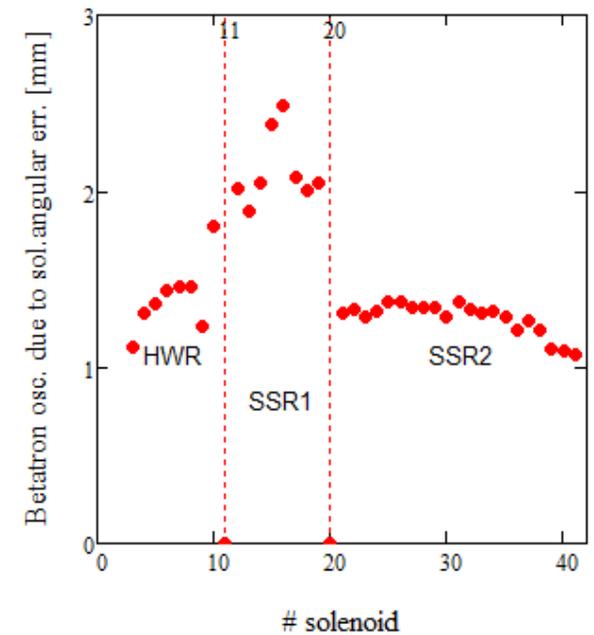
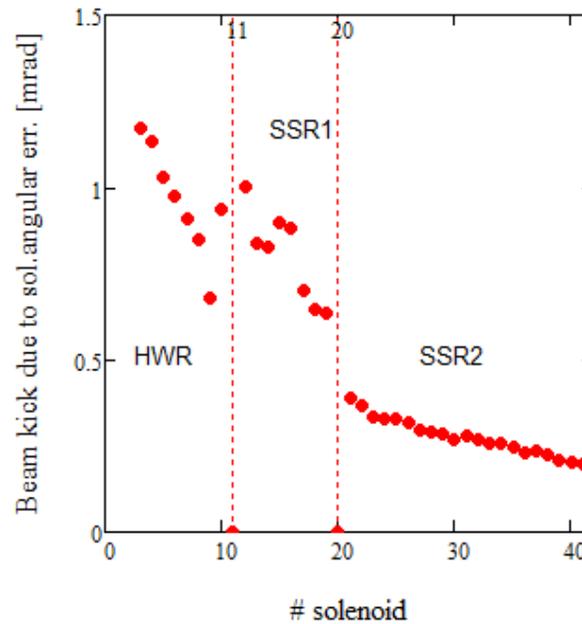
$$x_{eff} = \phi L_{sol} \left(2\sqrt{F / L_{sol}} \right)$$



Solenoid offsets corresponding to the angular errors presented in the FRSs: 1 mrad for HWR and SSR1 and 0.5 mrad for SSR2

Suggested changes to FRS

Beam kicks and betatron amplitudes due to angular errors of solenoids presented in the FRSs: 1 mrad for HWR and SSR1 and 0.5 mrad for SSR2



Before

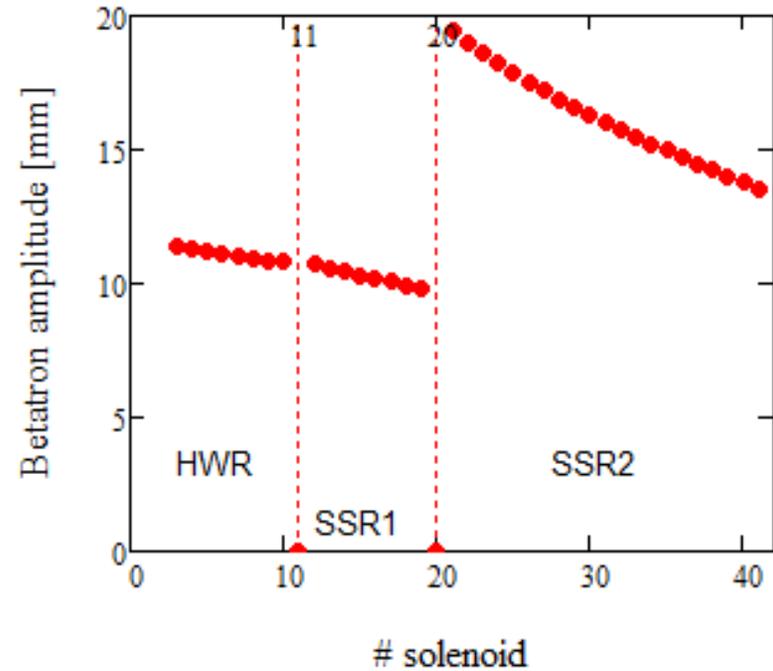
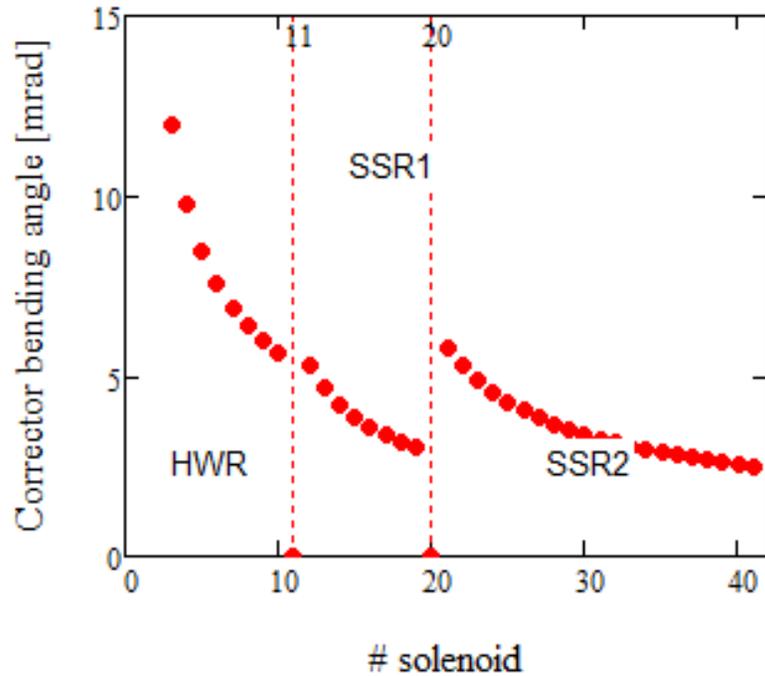
	HWR	SSR1	SSR2
Solenoid coordinate alignment, rms, mm	0.5	0.5	0.1
Solenoid angular alignment, θ , rms, mrad	1	1	0.5

After

	HWR	SSR1	SSR2
Solenoid coordinate alignment, rms, mm	0.5	0.5	0.5
Solenoid angular alignment, rms, mrad	1	0.5	0.5
Solenoid length, L_{sol} , mm	120	120	150
Offset due to angle, $L_{sol} \theta$, rms, μm	120	60	75

Requirements to the Strengths of Dipole Correctors

- Optics measurements require excitation of betatron amplitude of at least 3 mm by a single corrector



- Present correctors can excite significantly larger amplitude than minimally required

	HWR	SSR1	SSR2
Integral corrector strength, BdL, mT·m	2.5	2.5	5

Conclusions

- Required angular alignment requires measurements of solenoids ends to accuracy better than the requirements to offsets by an order of magnitude (500 μm versus $\sim 70 \mu\text{m}$, rms)
- Correctors should allow us a compensation of solenoid angular errors by about factor of 4
 - ◆ Thus the worst angular errors we can expect

	HWR	SSR1	SSR2
Solenoid coordinate alignment, rms, mm	0.5	0.5	0.5
Solenoid angular alignment, rms, mrad	4	2	2
Offset due to angle, $L_{\text{sol}} \theta$, rms, mm	0.5	0.25	0.3

- Critical consideration of solenoid alignment and has to be carried out