Limitations and Possible Improvements of the Booster Two-stage Collimation

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Fermilab

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- Common description of single and multiple scattering
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Objectives

- Major fraction of beam loss has to be absorbed inside collimation system
  - Scattering of protons out of second stage collimators has to be minimized
    - e.-m. scattering proceeds with small angles
      $\Rightarrow$ particles will be lost downstream of collimators
    - The goal should be >90% of protons experience nuclear interaction in the collimator joe
      $\Rightarrow$ Nuclear interaction results scattering with large angles which greatly reduces probability to scatter out of collimation system
- MARS + MADX are used for detailed simulations
- Simple analytical model helps to understand the process and limitations

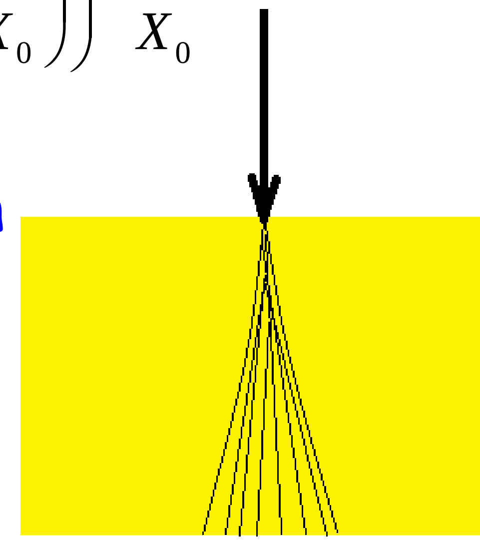
Multiple Scattering in a Medium

- Diffusion coefficient due to multiple scattering in a medium

$$D \equiv \frac{d}{ds}\overline{\theta_x^{\,2}} = \frac{4\pi n r_p^{\,2} Z(Z+1)}{\beta^4\gamma^2}\Lambda_c\ , \quad \Lambda_c = \ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right)$$

or from PDG: $$\overline{\theta_x^{\,2}} = \frac{1}{\beta^4\gamma^2}\left(\frac{13.6\,\mathrm{MeV}}{m_p}\right)^2\left(1+0.038\ln\left(\frac{s}{X_0}\right)\right)^2\frac{s}{X_0}$$

$\ln(s/X_0)$ appeared due to non-Gaussian tails



- Equation describing evolution of particle distribution

$$\frac{\partial f}{\partial s} + \theta\frac{\partial f}{\partial x} = \frac{D}{2}\frac{\partial^2 f}{\partial \theta^2}$$

- Consider the narrow beam hitting the medium

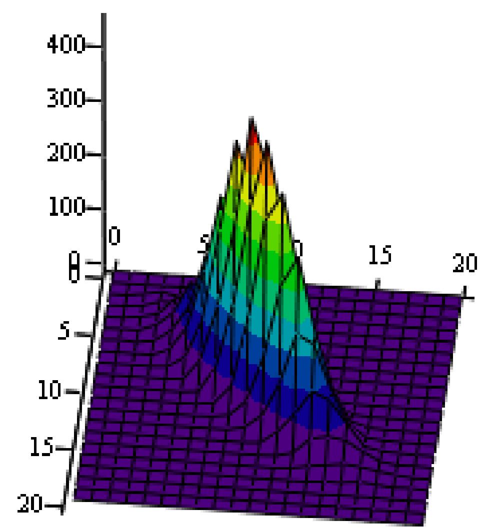
$$f(x,\theta) = \delta(x)\delta(\theta)$$

- The solution of diffusion equation is

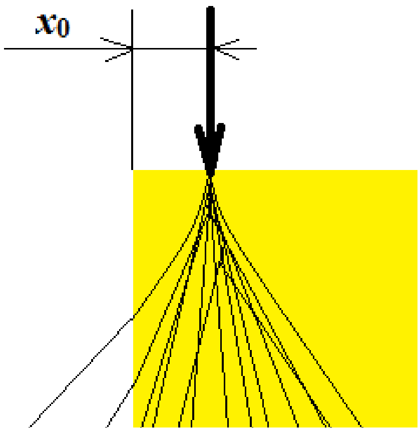
$$f_\delta(x,\theta,s) = \frac{\sqrt{3}}{\pi D s^2}\exp\left(-\frac{6x^2}{Ds^3} - \frac{6x\theta}{Ds^2} - \frac{2\theta^2}{Ds}\right)$$

$$\sigma_\theta = \sqrt{Ds}$$

$$\sigma_x = \frac{\sigma_\theta s}{\sqrt{3}} = \sqrt{\frac{Ds^3}{3}}$$



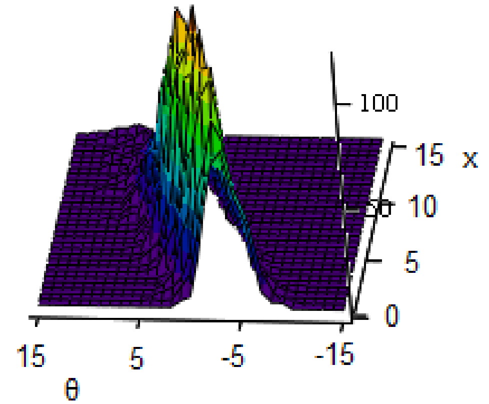
Out-scattering from Collimator



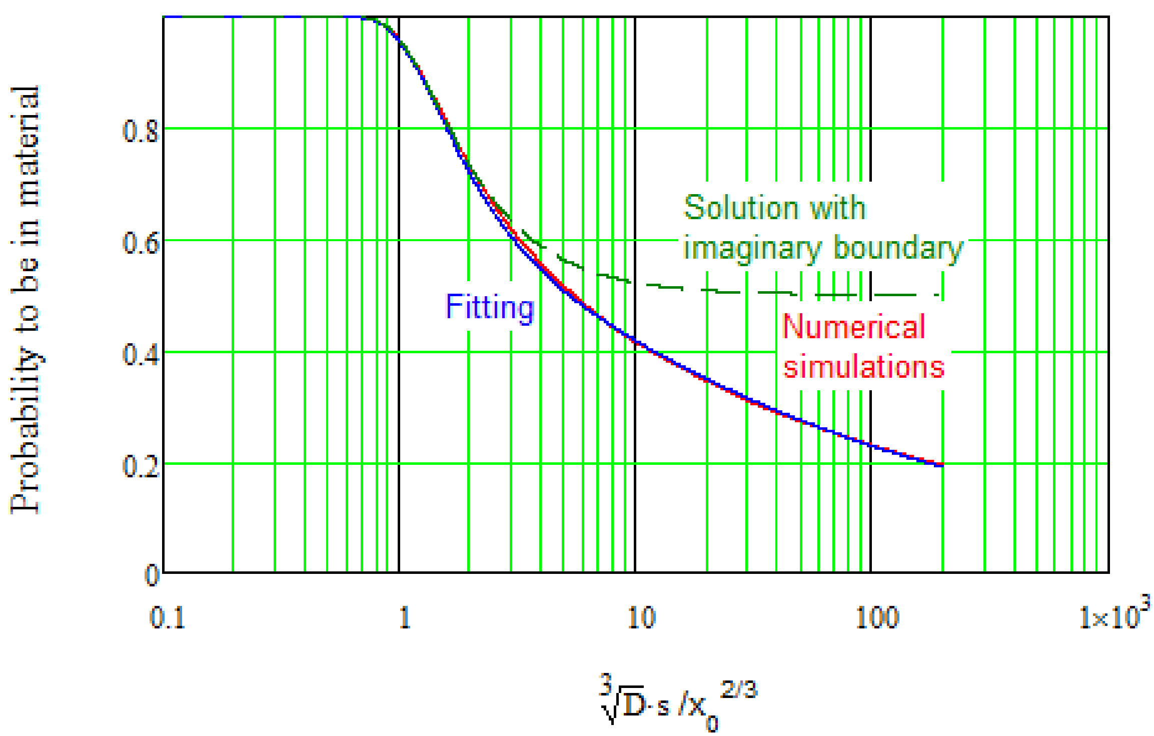
- Initial distribution $f(x,\theta) = \delta(x - x_0)\delta(\theta)$
- Boundary condition $f_\delta(x = 0, \theta > 0, s) = 0$
  - There is no particles incoming through the side
- A numerical solution can be parameterized

$$W(s) \equiv \frac{N(s)}{N_0} \approx 1 - \frac{1}{2}\left(erfc\left(\sqrt{\frac{3x_0^{\ 2}}{2Ds^3}}\right) + \exp\left(-\left(\frac{8^3 x_0^{\ 2}}{Ds^3}\right)^{2/9} - \left(\frac{8x_0^{\ 2}}{Ds^3}\right)^{2/27}\right)\right)$$

  - Parameterization accuracy is within ±2% for $\sqrt[3]{Ds^3 / x_0^{\ 2}} < 200$



  - The solution when we do not use the boundary condition (imaginary boundary) underestimates particle loss for $\sqrt[3]{Ds^3 / x_0^{\ 2}} > 2$

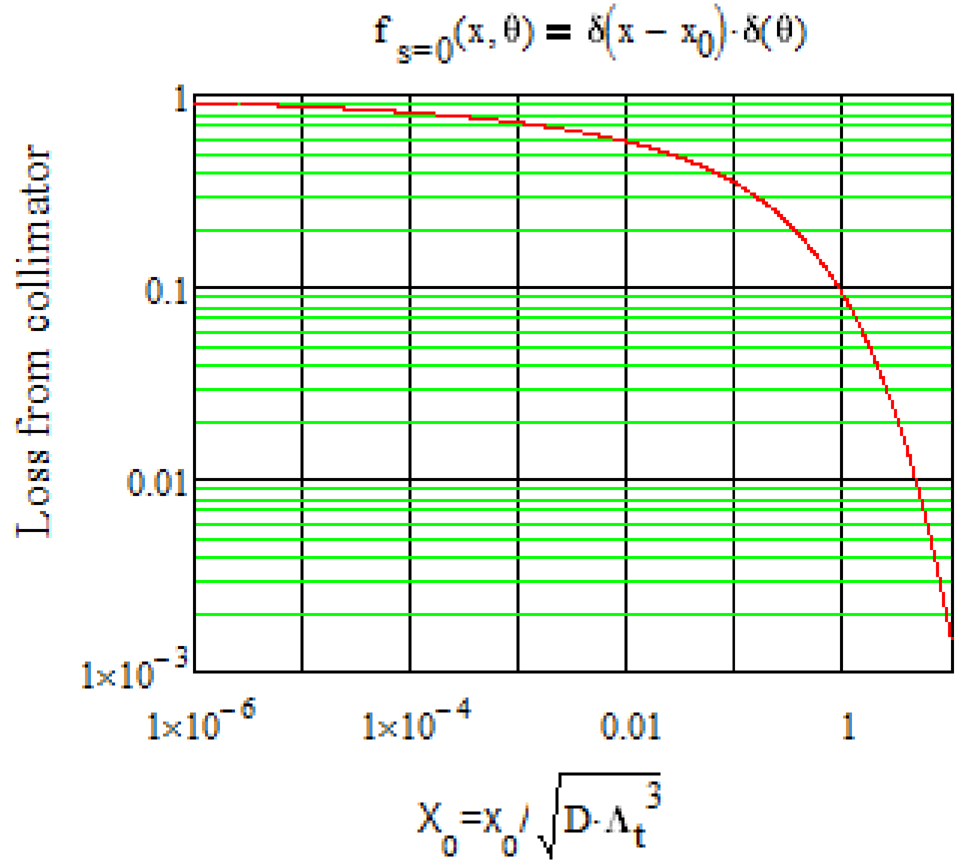


Out-scattering from Collimator for Narrow Beam

- Out-scattering from collimator due to multiple scattering results in particles with small angles which will get out of collimation system
- Nuclear interaction results large angular scattering or new particles with large angle which will stay in collimation system with high probability
  - Probability to scatter out before the first nuclear interaction characterizes efficiency of collimation

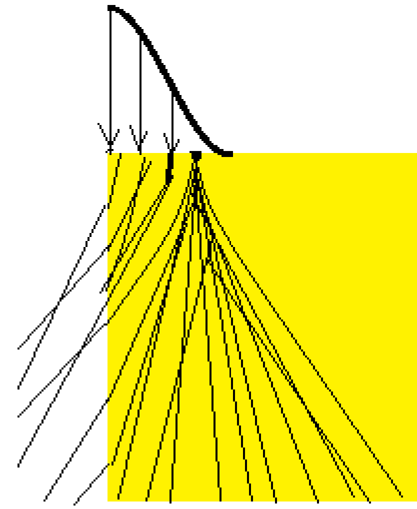
$$W_{loss}(x_0) = \int_0^\infty j(s) e^{-s/\Lambda_t} ds = \frac{ds}{\Lambda_t} \int_0^\infty W(s) e^{-s/\Lambda_t} \frac{ds}{\Lambda_t}, \quad j(s) = \frac{dW}{ds}$$

where Λ_t is the nuclear collision length



- Interception of 90% of particles requires $X_0 \approx 1$

Out-scattering from Collimator for Gaussian Beam

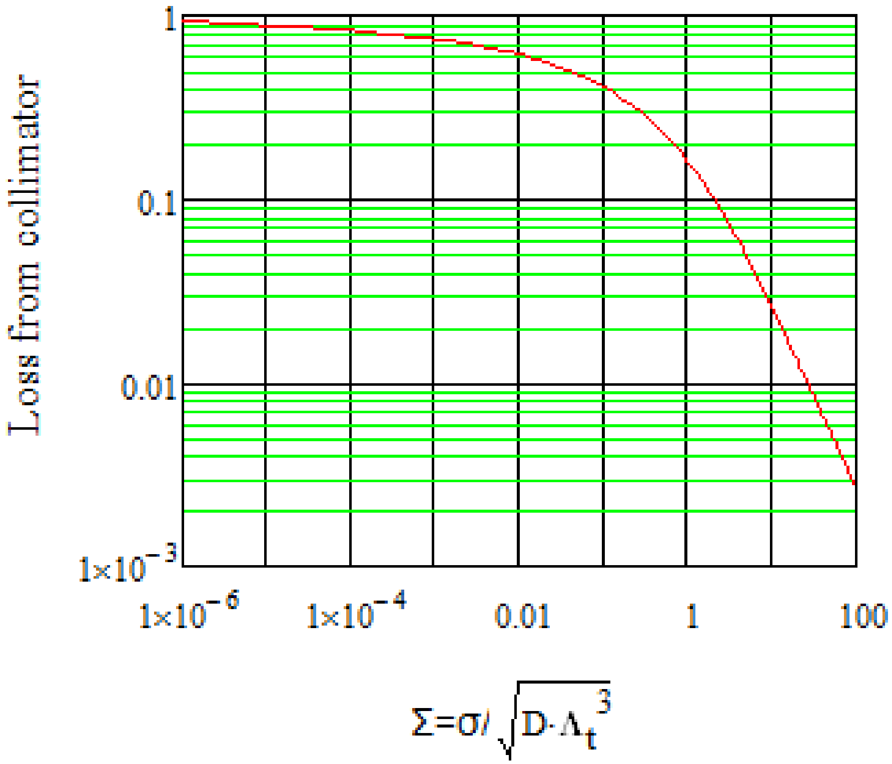


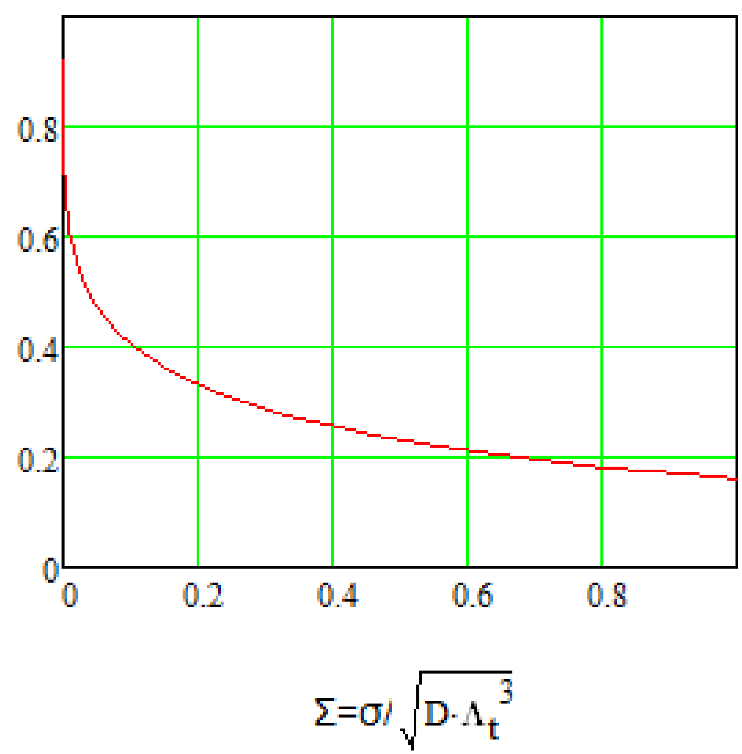
- For Gaussian beam, large number of particles is near the boundary
  - ⇨ Higher probability to scatter out

$$W_{lossG}(\sigma) = \sqrt{\frac{2}{\pi}} \int_0^\infty W_{loss}(x)\, e^{-x^2/2\sigma^2}\, dx$$

- Interception of 90% of particles requires

$$\Sigma = \sigma / \sqrt{D \Lambda_t^{\,3}} \approx 2.1$$





Common Description of Single and Multiple Scattering

- Scattering cross-section at small angles (large impact) parameters is limited by an atom size:

$$\frac{d\sigma}{d\theta_x} = \frac{4 r_p{}^2 Z^2}{\beta^4 \gamma^2} \frac{1}{\theta_x{}^3} \longrightarrow \frac{4 r_p{}^2 Z^2}{\beta^4 \gamma^2} \frac{1}{\left(\theta_x{}^2 + \theta_m{}^2\right)^{3/2}} \quad \Rightarrow \quad \sigma_{tot} = \int_{-\infty}^{\infty} \frac{d\sigma}{d\theta_x} d\theta_x = \frac{4\pi r_p{}^2 Z^2}{\beta^4 \gamma^2 \theta_m{}^2}$$

Value of θ_m is directly related to Coulomb logarithm

- Evolution of particle distribution is described by

$$\frac{\partial f}{\partial s} = n_a \int_{-\infty}^{\infty} f(\theta') \left(\left.\frac{d\sigma}{d\theta}\right|_{\theta-\theta'} - \sigma_{tot} \delta(\theta-\theta') \right) d\theta' = \frac{D}{\Lambda_c} \int_{-\infty}^{\infty} f(\theta') \left(\frac{1}{2\left((\theta-\theta')^2 + \theta_m{}^2\right)^{3/2}} - \sigma_{tot} \delta(\theta-\theta') \right) d\theta'$$

- Solution for $f|_{s=0} \approx \delta(\theta)$ is obtained using Fourier transform

$$f(\theta, s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp\left(-\frac{Ds}{\Lambda_c \theta_m{}^2} \left(1 - \kappa\theta_m K_1(\kappa\theta_m)\right) - i\kappa\theta \right) d\kappa$$

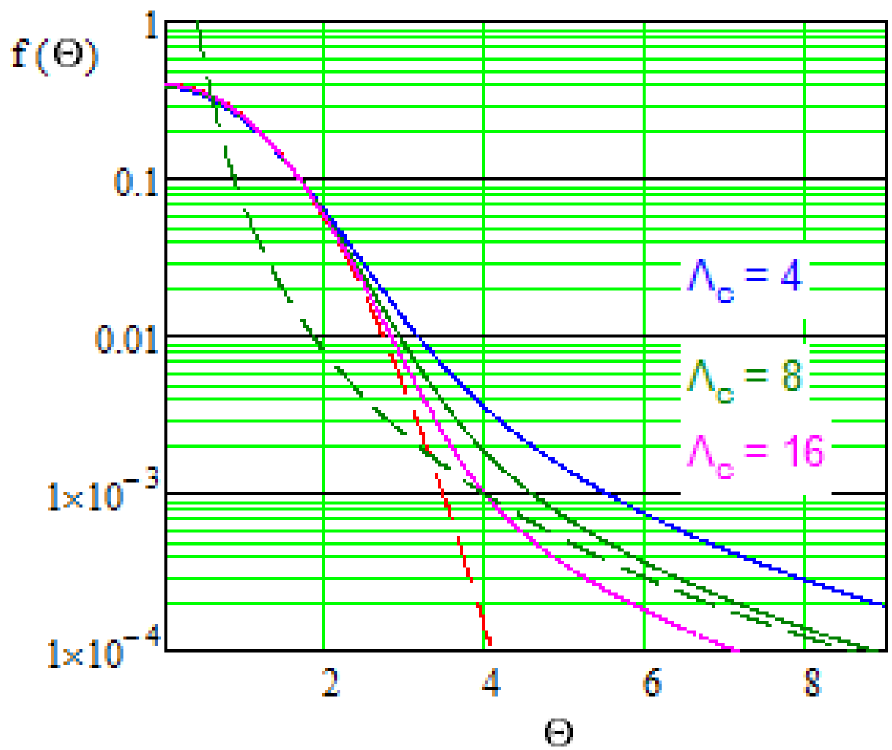
$$\xrightarrow{\theta_m \ll \sqrt{Ds}} \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp\left(-\frac{Ds\kappa^2}{2\Lambda_c} \ln\left(\frac{2}{e^{\gamma-1/2} \kappa\theta_m} \right) - i\kappa\theta \right) d\kappa \,, \quad \gamma \approx 0.577$$

- Matching the growth rate of distribution core to predictions of multiple scattering theory yields a relationship: $\Lambda_c \approx \ln\left(0.84\sqrt{Ds} / \theta_m\right)$

Common Description of Single and Multiple Scattering (2)

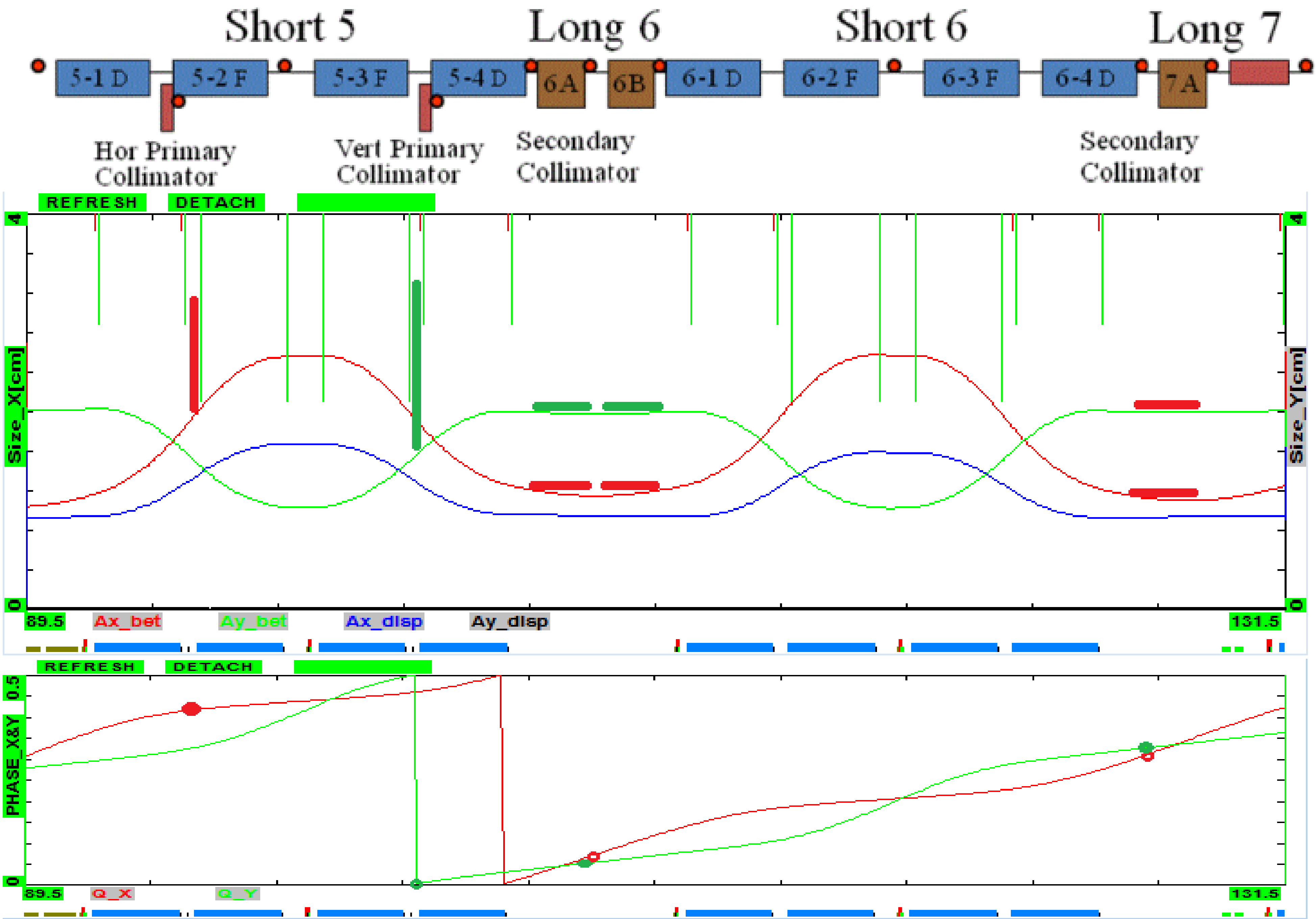
- With logarithmic accuracy the shape of the distribution does not depend on s:

$$f(\Theta) \approx \frac{1}{\pi}\int_0^\infty \cos(\Theta x)\exp\left(-\frac{x^2}{2\Lambda_c}\mathrm{n}\left(\frac{2.2}{xe^{-\Lambda_c}}\right)\right)dx\,, \quad \Theta = \frac{\theta}{\sqrt{Ds}}$$



- The distribution asymptote in tails is: $f(\Theta) \approx 1/\left(2\Lambda_c\Theta^3\right), \quad \Theta \gg 1$
- Tails include ~1% of particles and can be neglected in most cases

Limitations Coming from Booster Optics



Limitations Coming from Booster Optics

- To avoid particle loss between primary and secondary collimators the rms spot size on the secondary collimator due to scattering at the primary should not be larger than
  - X plane ~ 6 mm (15 mm/2.5σ=6 mm)
  - Y plane ~ 3 mm (8 mm/2.5σ=3.2 mm)
- That determines percentage of particles scattered out of collimators

Choice of Material for the Second Stage Collimators

- Material efficiency for second state collimation is characterized by the distribution transverse size at the nuclear collision length

$$\sigma_m = \sqrt{D \Lambda_t^{\,3}}$$

  - σ_m weakly depends on the material and is about 5 mm at 400 MeV
  - It is decreasing as $\propto 1/\left(\beta^2\gamma\right)$ with particle energy increase

Collimator materials and properties:

$$\begin{pmatrix} \mathrm{Al} \\ \mathrm{Ti} \\ \mathrm{Fe} \\ \mathrm{Ni} \\ \mathrm{Mo} \\ \mathrm{W} \end{pmatrix} \quad Z := \begin{pmatrix} 13 \\ 22 \\ 26 \\ 28 \\ 42 \\ 74 \end{pmatrix} \quad A := \begin{pmatrix} 26.98 \\ 47.87 \\ 55.8 \\ 56.7 \\ 95.95 \\ 183.8 \end{pmatrix} \quad \rho := \begin{pmatrix} 2.699 \\ 4.54 \\ 7.78 \\ 8.9 \\ 10.28 \\ 19.3 \end{pmatrix} \frac{\mathrm{g}}{\mathrm{cm}^3} \quad \lambda_T := \begin{pmatrix} 69.7 \\ 78.8 \\ 81.7 \\ 82.6 \\ 93 \\ 110.4 \end{pmatrix} \frac{\mathrm{g}}{\mathrm{cm}^2} \quad X_0 := \begin{pmatrix} 24.01 \\ 16.16 \\ 13.84 \\ 12.68 \\ 9.81 \\ 6.76 \end{pmatrix} \frac{\mathrm{g}}{\mathrm{cm}^2}$$

$$\begin{pmatrix} \mathrm{Al} \\ \mathrm{Ti} \\ \mathrm{Fe} \\ \mathrm{Ni} \\ \mathrm{Mo} \\ \mathrm{W} \end{pmatrix} \quad L_{col} := \overrightarrow{\frac{3\cdot\lambda_T}{\rho}} = \begin{pmatrix} 77.473 \\ 52.07 \\ 31.504 \\ 27.843 \\ 27.14 \\ 17.161 \end{pmatrix} \mathrm{cm} \qquad \overrightarrow{\left(1 + 0.038\cdot\ln\left(\frac{\lambda_T}{X_0}\right)\right)} = \begin{pmatrix} 1.04 \\ 1.06 \\ 1.067 \\ 1.071 \\ 1.085 \\ 1.106 \end{pmatrix}$$

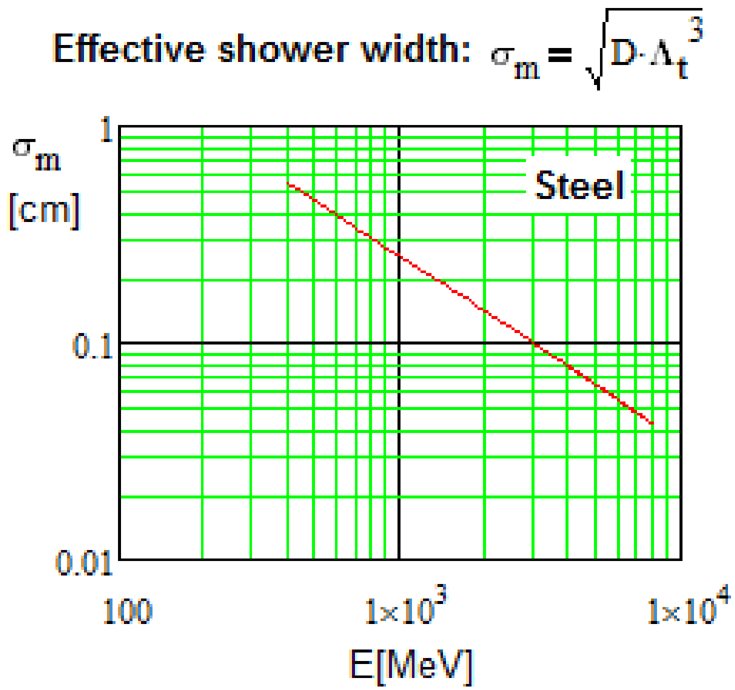
$$A_\theta := 13.6\cdot 10^6 \ \mathrm{eV}$$

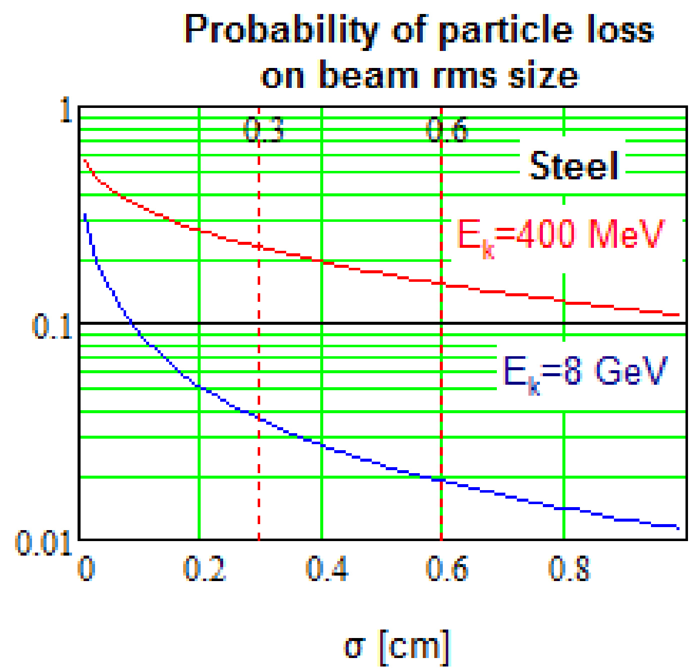
$$E_{in} := 400\cdot 10^6 \ \mathrm{eV} \qquad E_{fin} := 8\cdot 10^9 \ \mathrm{eV}$$

$$k := 0..100 \qquad E_k := E_{in} + \frac{E_{fin} - E_{in}}{100}\cdot k \qquad \gamma_k := 1 + \frac{E_k}{m_p} \qquad \beta_k := \sqrt{1 - \frac{1}{\left(\gamma_k\right)^2}}$$

$$\begin{pmatrix} \mathrm{Al} \\ \mathrm{Ti} \\ \mathrm{Fe} \\ \mathrm{Ni} \\ \mathrm{Mo} \\ \mathrm{W} \end{pmatrix} \quad \sigma_\theta := \overrightarrow{\left[\frac{A_\theta}{m_p\cdot\gamma_0\cdot\left(\beta_0\right)^2}\cdot\sqrt{\frac{\lambda_T}{X_0}}\cdot\left(1 + 0.038\cdot\ln\left(\frac{\lambda_T}{X_0}\right)\right)\right]} = \begin{pmatrix} 0.035 \\ 0.047 \\ 0.052 \\ 0.055 \\ 0.067 \\ 0.089 \end{pmatrix} \mathrm{rad} \qquad \sigma_m := \overrightarrow{\left(\frac{\lambda_T}{\rho}\cdot\sigma_\theta\right)} = \begin{pmatrix} 0.915 \\ 0.812 \\ 0.544 \\ 0.507 \\ 0.604 \\ 0.511 \end{pmatrix} \mathrm{cm}$$

Practical Estimates for Present Collimation Scheme

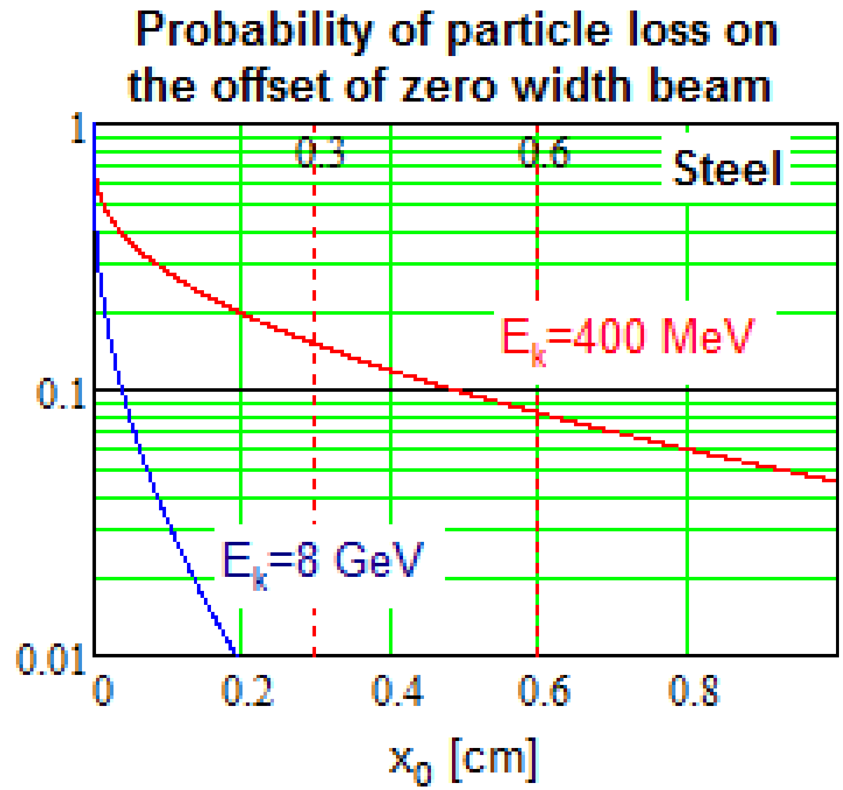




- Basing on the above theoretical estimates the theoretical limitation on particle loss from second stage collimation is about 25%
  - At Booster injection energy the loss will be worse because the ionization loss in the material will decrease particle energy and increase its scattering
- The major limitation is the limited Booster acceptance
  - It cannot be overcame in the existing paradigm of two stage collimation

Proposal for Collimation with High Efficiency

- We can replace the primary collimator by thin electrostatic septum
- The impact parameters at collimation system for particles lost at injection (e.g. due to space charge induced non-linearities) is close to 1 mm
  - ⇨ It means that the probability to hit a wire (30 μm diameter) is quite small. Scattering on the wires is also very small



- 90% collimation efficiency at injection requires an offset x_0~5 mm
  - ♦ Corresponding angular kick is ~0.3 mrad (β_f=15 m)
  - ♦ Required electric field integral is EdL~230 kV
    - L=15 cm, E=15 kV/cm, gap=7 mm, V=10 kV
  - ♦ Independent control for deflection in x&y will be greatly helpful

Conclusions

- Efficiency of existing two-stage collimator is limited to ~75%
  - In practice this value is limited to ~50%
- Using an electrostatic septum with moderate voltage and length (15 cm, 10 kV) should improve inefficiency by ~2 times
  - 90% efficiency should be the goal
- Adjustments of septum location and angle are required
  - Voltage ramp is not because kick values from septum and medium have the same dependence on energy
- Optimization with MARS simulations should exhibit what is achievable